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SOLUTIONS OF THE MODIFIED OSTROVSKII EQUATION WITH CUBIC NON-LINEARITY†

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Approximate solutions of the stationary Ostrovskii equation [1] with cubic non-linearity, describing, in particular, wave processes in rotating media, are constructed. A new integral invariant of this equation is found, making it possible to close the system of approximate equations and to eliminate excessive arbitrariness in the parameters. It is shown that there is a family of periodic stationary solutions in the form of a combination of a sawtooth wave with smoothed peaks and troughs, in which there are solitons of the modified Korteveg–de Vries equation of different polarity. This family of solutions is well described by approximate analytical theory, in which the role of a perturbation is essentially played by the ratio of the characteristic size of the soliton to the period of the sawtooth wave. The analytical solutions constructed are in good agreement with numerical solutions. The use of such solutions as the starting data for numerical calculations within the framework of the non-stationary equation has made it possible to establish the degree to which they are stationary and their stability to small perturbations.

The Ostrovskii equation has been investigated in many papers (see reviews [2, 3] and the references therein). Certain approximate stationary solutions [4] have been constructed (no accurate solutions are known), and the dynamics of individual non-stationary perturbations of the soliton type have been studied [5].

The approach used below to analyse the solutions of the modified Ostrovskii equation with cubic non-linearity is similar to that used previously for the related Ostrovskii equation with quadratic non-linearity [4, 5] and is based on matching two asymptotic branches of the solutions.

1. OSTROVSKII EQUATION AND ITS MODIFICATION

To describe long internal waves in a rotating ocean, Ostrovskii [1] derived the approximate equation

$$(v_t + \alpha v v_x + \beta v_{xxx})_x = f v \tag{1.1}$$

The coefficients α and β are determined by the structure of the eigenfunctions of the corresponding Sturm–Liouville boundary-value problem, while the coefficient $f > 0$ is proportional to the square of the Coriolis parameter.

Cases where the linear coefficient α in (1.1) vanishes (see, for example, [6, 7]) and it is necessary to take account of effects of the next order of infinitesimals with respect to linearity correspond to certain distributions of the density of the fluid in the ocean. Then, with an appropriate choice of scales, in place of (1.1), the modified Ostrovskii equation with cubic non-linearity arises:

$$(v_t + \alpha_1 v^2 v_x + \beta v_{xxx})_x = f v \tag{1.2}$$

If there is no right-hand side this equation reduces to the well-known modified Korteveg–de Vries equation, the coefficients of which (α_1 and β) are well known for internal waves (they can be found, for example, in [7]).

Equation (1.2) has many properties in common with Eq. (1.1). It can also be derived for waves in other media not associated with oceanology and fluid rotation, and therefore a study of possible types of solution and the construction of at least approximate stationary solutions are of interest.

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2. INTEGRALS OF MOTION

First we shall rewrite the equation in dimensionless form, using new normalized variables

$$t\left(288\frac{\beta f}{-\sigma}\,\frac{f^4}{\alpha_1^2\varepsilon^{10}}\right)^{1/4},\quad x_n = x\left(72\frac{-\sigma}{\beta f}\,\frac{f^4}{\alpha_1^2\varepsilon^6}\right)^{1/4},\quad u = v\left(\frac{1}{18}\,\frac{-\sigma}{\beta f}\alpha_1^2\varepsilon^2\right)^{1/4}$$

where $\sigma = 1$ if $\beta f < 0$ and $\sigma = -1$ if $\beta f > 0$. In these variables, Eq. (1.2) acquires the following form the subscript n is omitted below)

$$\left(u_t + 3u^2u_x - \frac{\sigma}{4}u_{xxx}\right)_x = \frac{\varepsilon^2}{2}u \tag{2.1}$$

where ε is a dimensionless parameter, and σ defines the type of dispersion in the short-wave region of the spectrum: when $\sigma = 1$ ($\sigma = -1$) the variance is positive (negative), i.e. the phase velocity of linear perturbations increases (decreases) as the wave number increases.

It can be shown that Eq. (2.1) possesses the following integrals of motion

$$I_1 \equiv \int u dx = 0,\quad I_2 \equiv \int u^2 dx = \text{const}$$

$$I_3 \equiv \frac{1}{4}\int\left(u^4 - \frac{\sigma}{2}u_\xi^2 + \varepsilon^2\varphi_\xi^2\right)d\xi = \text{const};\quad u = \varphi_{\xi\xi} \tag{2.2}$$

where the integration is carried out over a period of the wave λ for periodic perturbations or over the entire x axis for solitary waves. The third of the integrals has the meaning of the Hamiltonian, so that Eq. (2.1) can be written in the Hamiltonian form

$$u_t = (\delta I_3[u]/\delta u)_\xi \tag{2.3}$$

where $\delta/\delta u$ is a variational derivative of the functional $I_3[u]$.

Let us examine the family of stationary solutions of Eq. (2.1) of the form $u(x, t) = u(x - ct)$, where c is the velocity of the stationary wave. For solutions of this class, we obtain, in place of (2.1), a non-linear equation in ordinary derivatives

$$\left(-cu + u^3 - \frac{\sigma}{4}u''\right)'' = \frac{\varepsilon^2}{2}u \tag{2.4}$$

where the prime denotes differentiation with respect to the "running" coordinate $\xi = x - ct$.

This equation possesses a number of additional integral invariants which may be useful when investigating its solutions. In particular, assuming the solutions to be periodic (the absence of solitary stationary solutions was proved earlier [8]), we multiply Eq. (2.4) by the function $u(\xi)$ and integrate it over a wave period. As a result, we obtain the following integral of motion

$$I_4 \equiv \int\left(\frac{\varepsilon^2}{2}u^2 - cu'^2 + 3u'^2u^2 - \frac{1}{4}u''^2\right)d\xi = 0 \tag{2.5}$$

A further integral of Eq. (2.4) can be obtained if a new variable w, having the meaning of the "potential" for $u = w'$, is introduced, so that, in place of (2.4), we have

$$\left(-cw' + w'^3 - \frac{\sigma}{4}w'''\right)' = \frac{\varepsilon^2}{2}w \tag{2.6}$$

It can be shown that, in the solutions of this equation, the quantity

$$I_5 \equiv H = 2\varepsilon^2w^2 - \sigma w''^2 - 4cw'^2 + 2w'^4 \tag{2.7}$$

is retained.

The presence of this integral enabled us to reduce the initial differential equation to the second order. Since the variable ξ does not explicitly occur in this expression, its order can be reduced further. For this it is necessary to regard w as an independent variable, and w must be regarded as a function of w, i.e. $w' = F(w)$. Then, from (2.7) we obtain the first-order equation

$$F^2(\sigma F_w'^2 + 4c - 2F^2) - 2\varepsilon^2 w^2 + H = 0 \tag{2.8}$$

Some of the integrals found will be used below to construct approximate solutions.

3. STATIONARY SOLUTIONS

It is not possible to investigate the stationary solutions of Eq. (2.4), or of Eq. (2.8) stemming from it. Completely if , however, either the high-frequency dispersion is ignored, formally assuming $\sigma = 0$ in (2.4), or the low-frequency dispersion is ignored, assuming $\varepsilon = 0$, then the solutions of the reduced equation thus obtained can be investigated by simple means, for example by the phase-plane method. These solutions are of interest as asymptotic solutions of the complete Eq. (2.4). Moreover using item, it is possible to construct certain classes of approximate solutions, matching two such asymptotic solutions. The construction of approximate solutions of this kind will be examined in more detail.

We shall assume, first, that $\sigma = 0$, and find the first integral of the reduced equation (2.4):

$$(3u^2 - c)^2 u'^2 + \frac{\varepsilon^2 c}{2} u^2 - \frac{3\varepsilon^2}{4} u^4 = E_0, \quad E_0 = \text{const} \tag{3.1}$$

In this equation there is one state of equilibrium of the centre type (Fig. 1a), about which, in the phase plane, there are complete trajectories corresponding to smooth periodic solutions (Fig. 1b). All constrained solutions are contained in a rectangle with sides $2\sqrt{c/3}$ along the u axis and $2\varepsilon/\sqrt{12}$ along the u' axis (Fig. 1a). For motion along this rectangle in the phase plane there is a corresponding "limiting" periodic sawtooth wave

$$u_p(\xi) = \frac{\varepsilon}{\sqrt{12}}\left(-|\xi - m\lambda| + \frac{\lambda}{4}\right), \quad |\xi - m\lambda| \leqslant \frac{\lambda}{2}, \quad m = 0, \pm 1, \pm 2 \ldots \tag{3.2}$$

where λ is an arbitrary parameter, defining the wave period.

The maximum value of the wave (the "amplitude") $U_p = \varepsilon\lambda/\sqrt{12}$, and its velocity $c_p = 3U_p^2/16 = \varepsilon^2\lambda^2/64$.

If the low-frequency dispersion is now ignored ($\varepsilon = 0$), the first integral of Eq. (2.4) can be written in the form

$$\sigma u'^2 + 4u^2 - 2u^4 = E_1, \quad E_1 = \text{const} \tag{3.3}$$

In the phase plane (Fig. 2a) in this case there are three equilibrium positions and their type is determined by the sign of σ. Below, we shall confine ourselves to the case where $\sigma = -1$, since it is only in this case that Eq. (2.8) possesses soliton solutions, which are necessary subsequently to construct

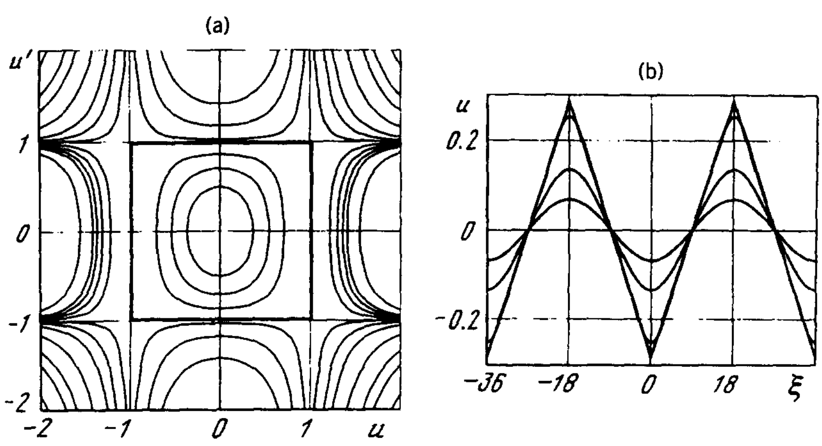

Fig. 1.

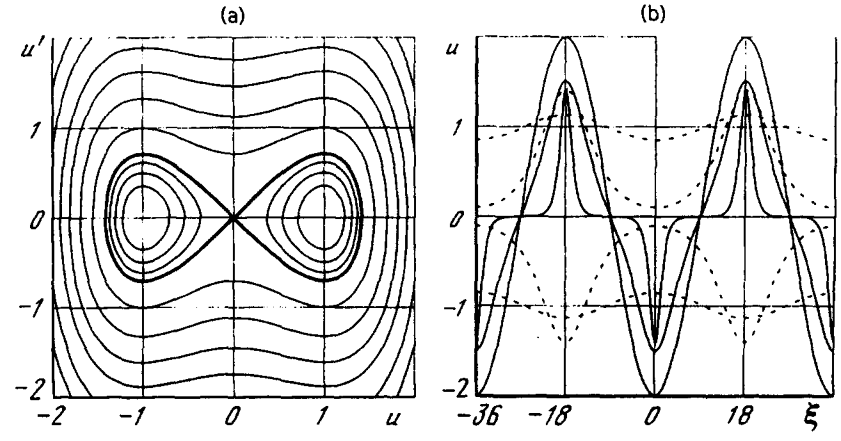

Fig. 2.

approximate solutions of the complete equation (2.4). Under this condition, one of the equilibrium states is a saddle point, while the other two are centres. All trajectories are bounded and complete. Close to the centres, the trajectories describe non-linear periodic waves with a non-zero average value and an asymmetrical profile: sharp peaks and shallow troughs or, conversely, shallow peaks and sharp troughs (the dashed curves in Fig. 2b). The asymmetry is more pronounced the closer the trajectory comes to the separatrix, shown by the bold line in Fig. 2(a). "Limiting" solutions again correspond to motions over the separatrix, but of a different kind—modified Korteveg–de Vries solitons of positive and negative polarity. Their shape is well known and is easily found from relation (3.3) with $E_1 = 0$

$$u_s(\xi) = \frac{A}{\mathrm{ch}(\xi/\Delta)}, \quad c_s = \frac{A^2}{2}, \quad \Delta = \frac{1}{|A|\sqrt{2}} \tag{3.4}$$

Finally, complete trajectories outside the separatrix the (continuous curves in Fig. 2b) describe periodic perturbations of symmetrical shape, which at relatively low amplitudes are a sequence of heteropolar solitons, and at large amplitudes quasisinusoidal waves.

The complete equation (2.4) possesses more complex stationary solutions which, in the general case, cannot be represented in the phase plane, but, for certain values of the coefficients of Eq. (2.4), their shape may be similar to a combination of individual parts of the solutions mapped in Figs 1(b) and 2(b). The family of these solutions can be constructed by matching two limiting asymptotic solutions: sawtooth wave (3.2) and the modified Korteveg–de Vries soliton (3.4).

The following physical reasoning will indicate the possibility of such solutions. We will consider smooth periodic perturbations of the wave λ and make a rough estimate of the dispersion terms in Eq. (2.4). As regards order of magnitude, an estimate of the high-frequency dispersion gives $u/(4\lambda^4)$, whereas for low-frequency dispersion we have $\varepsilon^2 u/2$. From this it can be seen that, for sufficiently large λ, we can ignore high-frequency dispersion compared with low-frequency dispersion and examine reduced Eq. (3.1) and the solutions corresponding to it. Incidentally, note that the "limiting solution" (3.2) of this equation also satisfies the complete equation (2.4) in almost all cases, with the exception of a denumerable number of-inflection points of the profile at which the derivative has not been determined. It is at these points that high-frequency dispersion becomes predominant, so that low-frequency dispersion can now be ignored. Then, in the vicinity of the points of inflection, complete equation (2.4) reduces to reduced equation (3.3), the solution of which may be, in particular, modified Korteveg–de Vries solitons (3.4). Therefore, we can assume the existence of periodic solutions of Eq. (2.4), the profile of which in almost all cases is similar to sawtooth wave (3.2), while at the peaks and troughs there are narrow modified Korteveg–deVries solitons with a characteristic size Δ much smaller than the period of the wave λ.

4. APPROXIMATE ANALYTICAL SOLUTIONS

Hence, proceeding with the construction of the approximate solutions of Eq. (2.4), we shall distinguish two scales: an inner scale of order Δ and an outer scale of order λ. The inner solution of Eq. (2.4) will be represented in the form of an asymptotic series, assuming the parameter ε to be small.

$$u(\xi) = u_0(\xi) + \varepsilon u_1(\xi) + \varepsilon^2 u_2(\xi) + \dots$$
$$c = c_0 + \varepsilon c_1 + \varepsilon^2 c_2 + \dots \tag{4.1}$$

When $\varepsilon = 0$, the given solution is converted into an exact modified Korteveg–de Vries solution [9] stemming from Eq. (2.4) with zero right-hand side. Modified Korteveg–de Vries soliton solution (3.3) will be selected as the zero approximation of $u_0(\xi)$. We substitute expansion (4.1) into Eq. (2.4) and write the equations obtained for first and second approximations with respect to the parameter ε. Integrating them twice, we obtain

$$\frac{u_1''}{4} + \frac{u_1}{4\Delta^2} + 3u_0^2 u_1 = c_1 u_0 + \tilde{A}_1 \xi + \tilde{B}_1$$
$$\frac{u_2''}{4} + \frac{u_2}{4\Delta^2} + 3u_0^2 u_2 = c_2 u_0 + c_1 u_1 - 3u_0 u_1^2 + \Delta A \int \mathrm{arctg}[\exp(\xi/\Delta)] d\xi + \tilde{A}_2 \xi + \tilde{B}_2 \tag{4.2}$$

where $\tilde{A}_1, \tilde{A}_2, \tilde{B}_1$, and $\tilde{B}_2$ are integration constants. Far from the soliton peak, for sufficiently large ξ, the equations are simplified due to the fact that the function $u_0(\xi)$ approaches zero exponentially rapidly. This produces simple linear equations of the second order with polynomial right-hand sides.

$$u_1'' + \frac{u_1}{\Delta^2} = A_1 \xi + B_1$$
$$u_2'' + \frac{u_2}{\Delta^2} = 4c_1 u_1 + A_2 \xi + B_2 \tag{4.3}$$

where A_1, A_2, B_1, and B_2 are new integration constants. In the second equation of system (4.3), we have taken into account the fact that, for sufficiently large ξ, the function arctg $[\exp(\xi/\Delta)] \approx \pi/2$, while the integral from it is proportional to ξ. Particular solutions of inhomogeneous Eqs (4.3) have the form

$$u_1(\xi) = \Delta^2 (A_1 \xi + B_1)$$
$$u_2(\xi) = \Delta^2 [(4c_1 A_1 \Delta^2 + A_2)\xi + 4c_1 B_1 \Delta^2 + B_2] \tag{4.4}$$

From these solution it can be seen that the function $u_1(\xi)$, just like $u_2(\xi)$, is linear with respect to ξ. Corrections to the velocities c_1, and c_2, remain unspecified. Thus, taking the zero approximation into account, the required solution, up to terms in ε^2, can be written in the form

$$u(\xi) \approx \frac{A}{\mathrm{ch}(\xi/\Delta)} + \varepsilon \Delta^2 (4A_1 \xi + B_1) + \varepsilon^2 \Delta^2 [(4c_1 A_1 \Delta^2 + A_2)\xi + 4c_1 \Delta^2 B_1 + B_2] \tag{4.5}$$

The outer solution of Eq. (2.4) will be sought in the form

$$u = \varkappa \xi + \mu \tag{4.6}$$

Substituting this expression into Eq. (2.4), it can be shown that (4.6) is its exact solution for $\varkappa^2 = \varepsilon^2/12$ and any μ. This means that the solutions of Eq. (2.4) may be linear functions both with positive angular coefficients $\varepsilon/\sqrt{12}$, and with negative angular coefficients $(-\varepsilon/\sqrt{12})$. Along the whole straight line ξ, such solutions, obviously have no physical meaning in view of their unbounded nature, but on individual segments they can be quite acceptable. For example, it is possible to construct from them periodic sawtooth wave (3.2), which in all cases, with the exception of points of inflection of the profile (peaks and troughs), will satisfy Eq. (2.4). The presence of the higher derivatives in Eq. (2.4) should lead to smoothing of the wave profile. Such smoothing can be carried out approximately by means of solution (3.4), placing solitons of negative polarity in the troughs, as shown in Fig. 3(a) by the continuous bold curve.

Comparing the behaviour of the solution far from the points of inflection and accordingly far from the peaks and troughs of the solitons, we reach the conclusion that in formula (4.5) it is necessary to leave only linear terms in ε, while the quadratic terms in ε must vanish by an appropriate choice of constants. Thus, we obtain

$$A_1 = A^2/\sqrt{3}, \quad A_2 = -4c_1 A_1 \Delta^2$$
$$B_1 = \mp \lambda A^2/(4\sqrt{3}), \quad B_2 = -4c_1 B_1 \Delta^2 \tag{4.7}$$

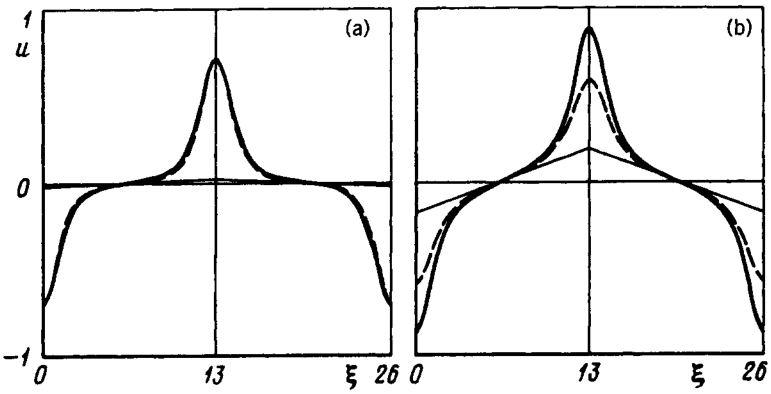

Fig. 3.

The upper sign relates to the solution to the right of the peak of the soliton placed at the origin of coordinates, and the lower sign relates to the solution to the left. Here, c_1 remains an undetermined parameter. The solution constructed in this way in the interval $(-\lambda/2, \lambda/2)$ continues periodically along the entire ξ axis.

It can be seen that, in higher orders in ε, linear solutions of $u_n(\xi)$ will again occur, containing arbitrary coefficients A_n, and B_n. All these higher corrections should also vanish by suitable choice of the constants A_n and B_n.

At this stage, the solution contains three free parameters: the amplitude of the soliton A, the wavelength λ and the correction to the velocity c_1. However, account must be taken of the constraint associated with the integral invariant I_4. Substituting the solution obtained in the form

$$u(\xi)=\begin{cases}\dfrac{A}{\operatorname{ch}(\xi/\Delta)}+\dfrac{\varepsilon}{\sqrt{12}}\left(-|\xi|+\dfrac{\lambda}{4}\right), & |\xi|\leq\dfrac{\lambda}{4}\\ \dfrac{-A}{\operatorname{ch}[(\xi-\operatorname{sign}(\xi)\lambda/2)/\Delta]}+\dfrac{\varepsilon}{\sqrt{12}}\left(-|\xi|+\dfrac{\lambda}{4}\right), & \dfrac{\lambda}{4}\leq|\xi|\leq\dfrac{\lambda}{2}\end{cases} \tag{4.8}$$

after elementary calculations we find the correction to the velocity c_1 (here, we used the fact that the wavelength is much greater than the characteristic size of the soliton). Then, the velocity of the wave is finally equal to

$$c\approx\frac{A^2}{2}+\varepsilon\left(-\frac{7}{8}\sqrt{\frac{3}{2}}-\frac{3}{4}\sqrt{\frac{3}{2}}G-\frac{3}{2A}\sqrt{\frac{3}{2}}+\frac{3\sqrt{3}\pi A\lambda}{64}\right) \tag{4.9}$$

where $G \approx 0.916$ is Catalan's constant [10]

Thus, the solution constructed formally contains two independent parameters: the soliton amplitude A and the wavelength λ, However, by virtue of the assumptions made, the soliton amplitude (the zero approximation) should be much greater than the amplitude of a sawtooth wave, i.e. the following condition should be satisfied.

$$A\sqrt{12}/(\varepsilon\lambda)\gg 1 \tag{4.10}$$

An example of a combined periodic solution of this kind, consisting of a sawtooth periodic wave with smoothed peaks and troughs in which there are modified Korteveg–de Vries solitons of different polarity, is shown in Fig. 3(a) by the continuous bold curve. Here, $A\sqrt{12}/(\varepsilon\lambda) = 9.3$.

Note that, when constructing similar periodic solutions for Ostrovskii's equation with quadratic nonlinearity [4], we used integral invariant I_1 (2.2) which, in the case examined here, is identically equal to zero by virtue of the antisymmetry of the solutions constructed. Therefore, the integral invariant I_4 (2.5) found here has turned out to be very opportune for eliminating any arbitrariness in the wave velocity.

5. NUMERICAL SOLUTIONS

Stationary solutions of Eq. (2.4) can also be constructed numerically by Petviashvili's method [11]. This enables is to compare the approximate analytical solutions with the numerical solutions, and also to study the structure of periodic solutions outside the area of applicability of the asymptotic theory. This approach was used in a study of the properties of Ostrovskii's equation with a quadratic non-linearity [4].

Using the program developed earlier, after slight correction, a numerical search for stationary periodic solutions of Eq. (2.4) was made. Assuming the coefficients of Eq. (2.4) are known (assuming, in particular, that $\sigma = -1$ and $\varepsilon = 0.01$), we shall set the period of the wave $\lambda = 26$ and the soliton amplitude $A = 0.7$. Then, by (4.9), the theoretical wave velocity $c \approx 0.26$. The profile of a wave with such parameters, representing the approximate solution, described by formula (4.8), is represented in Fig. 3(a) by the continuous bold curve. A wave of sawtooth profile with the same values of the parameters, described by formula (3.2), is represented in Fig. 3(a) by the thin continuous line. The numerical solution constructed by Petriashvili's method with the same values of ε, λ and A is shown in Fig. 3(a) by the dashed bold curve. It can be seen that the approximate theoretical solution is very similar to the numerical solution for fairly small ε. For such values of the parameters, condition (4.10) $A\sqrt{12}/(\varepsilon\lambda) \approx 9.3 \gg$ is satisfied quite well, which corresponds to the limits of applicability of the theory constructed here. However, when the parameter ε increases, condition (4.10) begins to be violated, and therefore the quantitative differences between the approximate and numerical solutions are increasingly pronounced, although qualitative agreement between them is retained.

Figure 3(b) shows a comparison of the two types of solution when $\varepsilon = 0.1$ and the previous values of the remaining parameters ($\lambda = 26$ and $A = 0.7$). The profile of a wave with such parameters, constituting an approximate theoretical solution, is represented by the continuous bold curve, a wave of sawtooth profile with the same values of the parameters is represented by the continuous thin line, and the numerical solution is shown by the dashed bold curve. Here, the theoretically obtained wave velocity $c \approx 0.255$. Condition (4.10) gives $A\sqrt{12}/(\varepsilon\lambda) \approx 9.3$, which is already beyond the limit of applicability of the theory constructed here.

6. THE STABILITY OF THE APPROXIMATE STATIONARY SOLUTIONS

To check the extent to which the solutions constructed are stationary and their stability to perturbations, evolution equation (1.2) was solved numerically. Approximate analytical solution (4.8) was used as the initial conditions.

For the solution shown in Fig. 3(a) by the continuous bold curve ($\varepsilon = 0.01$; $\lambda = 26$; $A = 0.7$), which is in good agreement with the limits of applicability of the theory, in the numerical calculation, stationary displacement occurred along the x axis for a long period of time, and no tendency towards its breakdown was observed. However, the numerically obtained velocity of wave propagation (≈ 0.28) proved to be

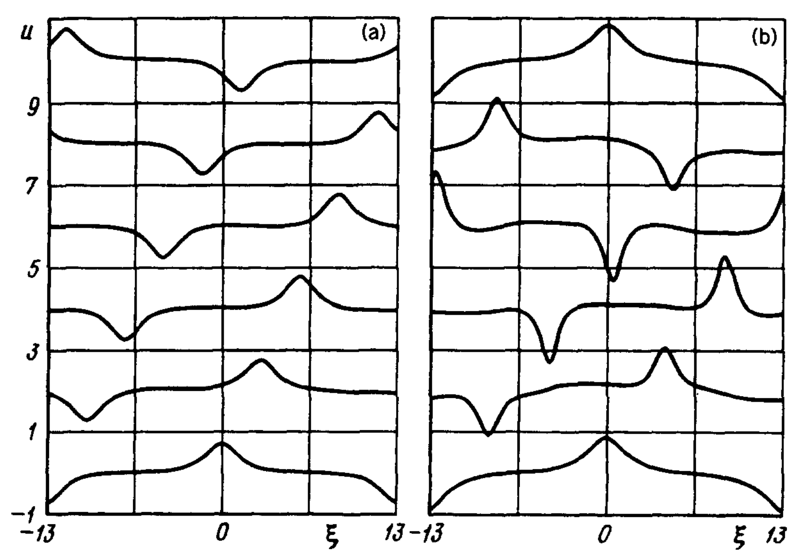

Fig. 4.

slightly higher than the theoretical velocity (≈ 0.26), and here the velocity of a modified Korteveg–de Vries soliton with the same amplitude was 0.245, and the correction to the velocity was ≈ 0.01. It was found that the difference between numerical and theoretical values of wave velocity decreases as the parameters ε and λ decrease. The shape of the stationary solution, constructed numerically by Petviashvili's method, again remained stationary, its velocity agreeing with the theoretical value.

Figure 4(a) presents the long term evolution of the numerical solution at different instants of time with approximate initial condition (4.8) and the values of the parameters indicated above. The vertical scale only applies to the lowest curve ($t = 0$), and all remaining curves are displaced successively along the vertical. As can be seen, in this case the approximate solution actually gives rise to a steady travelling wave.

Figure 4(b) shows the time evolution of the initial perturbation, the profile of which is represented in Fig. 3(b) by the continuous bold curve (here $\varepsilon = 0.1$; $\lambda = 26$; $A = 0.7$). The vertical scale again only applies to the lowest curve ($t = 0$), and all the remaining curves are displaced successively along the vertical. Since the conditions of applicability of the theory with these parameters are not satisfied, extremely non-stationary wave evolution is observed here. In this case, as the wave propagates, marked distortions of its shape occur, but, after a certain time, roughly equal to the wave period, its shape returns to its original form. Such a recurrence effect is well known for systems close to being completely integrable [9].

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